If $p>n$, then, since there can be but n different ways of voting, n will be the number of different ways the voting may result.

If $p<n$, then since p persons can prepare only p states of the poll, p will be the number of different ways the voting may result.

Also solved by *H. C. Whitaker.*

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

40. Proposed by J. C. CORBIN, Pine Bluff, Arkansas.

If R, r, r_1, r_2, and r_3 be, respectively, the radii of the circumscribed, inscribed, and escribed circles of a $\triangle$, prove $r_1+r_2+r_3-r=4R$.

Solution by M. A. GRUBER, War Department, Washington, D. C.

From any $\triangle$ whose sides are a, b, and c, we obtain $R=\dfrac{abc}{4\triangle}$,

$$r=\frac{\triangle}{s},\ r_1=\frac{\triangle}{s-a},\ r_2=\frac{\triangle}{s-b},\ \text{and } r_3=\frac{\triangle}{s-c}.$$

$$\therefore\quad r_1+r_2+r_3-r=\frac{\triangle}{s-a}+\frac{\triangle}{s-b}+\frac{\triangle}{s-c}-\frac{\triangle}{s}=\frac{2s^3-as^2-bs^2-cs^2+abc}{\triangle}$$

$$=\frac{s^2[2s-(a+b+c)]+abc}{\triangle}=\frac{abc}{\triangle}.\quad \text{But } \frac{abc}{\triangle}=4R.\quad \therefore\quad r_1+r_2+r_3-r=4R.$$

We might appropriately add a few other combinations of these radii.

$$(1)\ \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r};\ (2)\ rr_1r_2r_3=\triangle^2;\ (3)\ Rrr_1r_2r_3=\frac{abc\triangle}{4}.$$

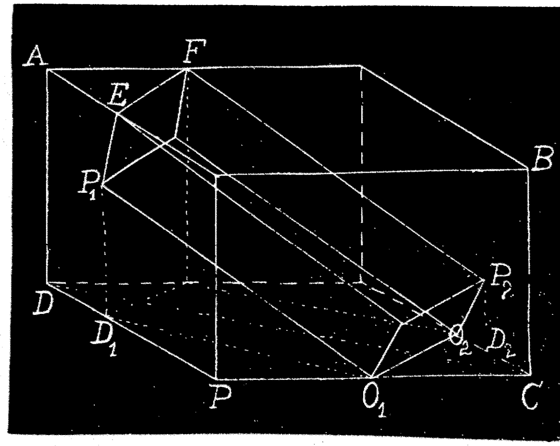
Solutions of this problem were received from *G. I. Hopkins*, *E. W. Morrell*, *P. S. Berg*, *G. B. M. Zerr*, *F. P. Matz*, *Cooper D. Schmitt*, *P. H. Philbrick*, *J. F. W. Scheffer*, *John B. Faught*, and the *Proposer*. H. C. Whitaker did not solve the problem but referred to Chauvenet's Geometry and Hallowell's Geometrical Analysis, p. 225.

41. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find the length (x) of a rectangular parallellopiped $b=5$ft., and $h=3$ft., which can be *diagonally inscribed* in a similar parallelopiped $L=83$ft., $B=64$ft, and $H=50$ft.

Solution by B. F. BURLESON. Oneida, Castle, New York, and the PROPOSER.

Let $x=AF$, $y=AE$, $z=P_2D_2$, and $l=O_1P_1$ =the required length of the inscribed rectangular parallelopiped; then, obviously, $x^2+y^2=b^2$ ....(1), $(L-x)^2+(B-y)^2+(H-z)^2=l^2$ ....(2), $x(L-x)=y(B-y)$ .....(3), and $h\sqrt{[(L-x)^2+(B-y)^2]}=lz$ ....(4). From (3) and (1), $4y^4-4By^3+(B^2-4b^2+L^2)y^2+2Bb^2y=(L^2-b^2)b^2$ ....(5); and this with coefficients numerically expressed, becomes $4y^4-256y^3+10885y^2+3200y=171600$ .....(6). Therefore, by *Horner's Method of Approximation*, we have from (6), $y=4$; whence $x=3$. Briefly putting the now known $(L-x)^2+(B-y)^2,=m^2=10000$, we have from (2) and (4), respectively, $m^2+(H-z)^2=l^2$ ....(7), and $lz=hm$....(8). Therefore, $l^4-(H^2+m^2)l^2+2Hhml=h^2m^2$ ....(9); that is, $l^4-12500l^2+30000l=90000$ ....(10).



Whence $l=110.617130324415$ feet.

COR.—Make $H=0$, and $h=0$; then the problem becomes: *Find the length of a rectangle of given width inscribed diagonally in a given rectangle.*

After performing obvious operations, we obtain

$l^4-(B^2+2b^2+L^2)l^2+4BbLl=(B^2-b^2+L^2)b^2$ ....(11); or with the coefficients numerically expressed, we have the equation,

$$l^4-11035l^2+106240l=274000 \ldots.(12).$$

Therefore $l=100$ feet, which is the length of the diagonally-inscribed rectangle required.

A. H. Bell gets 107.5 feet as a result.

42. Proposed by G I. HOPKINS, Instructor in Mathematics and Physics in High School, Manchester, New Hampshire.

If the bisectors of two angles of a triangle are equal the triangle is isosceles.

Solution by the PROPOSER.

Let AF and BD bisect the angles of the triangle ABC, and let $AF=BD$.

Draw DE. Make $\angle PDO=\angle PDF$, and $\angle QFN=\angle QFD$.

Draw AH perpendicular to AF and BK perpendicular to BD.

Draw FH through O and DK through N.